Six Functions of a Dollar

Made Easy!

Six Functions of a Dollar

Here's a list.

Simple Interest

Future Value using Compound Interest

Present Value

Future Value of an Annuity

Present Value of an Annuity

Amortization of Future Value (aka Sinking Fund Factor)

Amortization of Present Value (aka Partial Payment Factor)

Wait a minute!!!!!!!
That's **Seven** Functions, not Six.

Easy Explanation: Simple Interest is not considered one of the six functions.

Then why is it listed!!!!!!!

Most college finance textbooks begin with Simple Interest as a starting point for teaching Future Value where interest is compounded.

Are we good now?

O.K. Rock On!!!!!!!

Simple Interest

$$FV = PV(1 + in) = PV(1 + i)n$$

$FV =$ *Future Value*

$PV =$ *Present Value*

$i =$ *Interest Rate (eg Monthly or Annual)*

$n =$ *Number of periods (eg months or years)*

Notice with Simple Interest no interest is earned on prior year interest. Interest is earned only on the original principal.

ALWAYS be sure to match the interest rate period to that used in the number of periods. In other words, do not use an annual interest rate and a period expressed in months.

EXCEL CLASS EXAMPLE: Use Excel to calculate the following future values:

Initial Investment $20,000
Monthly Interest Rate is 0.3%
Compare periods of 6 months, 12 months, 24 months, 60 months, & 240 months

Future Value Using Compound Interest

$$FV = PV(1+i)^n$$

$FV =$ alue

$PV =$ 'alue

$i =$ ate (eg Monthly or Annual)

$n =$ ' periods (eg months or years)

Notice with Compound Interest interest is earned on prior year interest. Interest is earned not only on the original principal but also on interest earned during the course of the investment.

ALWAYS be sure to match the interest rate period to that used in the number of periods. In other words, do not use an annual interest rate and a period expressed in months.

EXCEL CLASS EXAMPLE: Use Excel to calculate the following future values:

Initial Investment $20,000
Monthly Interest Rate is 0.3%
Compare periods of 6 months, 12 months, 24 months, 60 months, & 240 months

THEN compare the results with your earlier Excel simple interest calculations.

Present Value Using Compound Interest

DERIVATION:

Start With Future Value Formula $FV = PV(1+i)^n$

$$\frac{FV}{(1+i)^n} = \frac{PV(1+i)^n}{(1+i)^n}$$

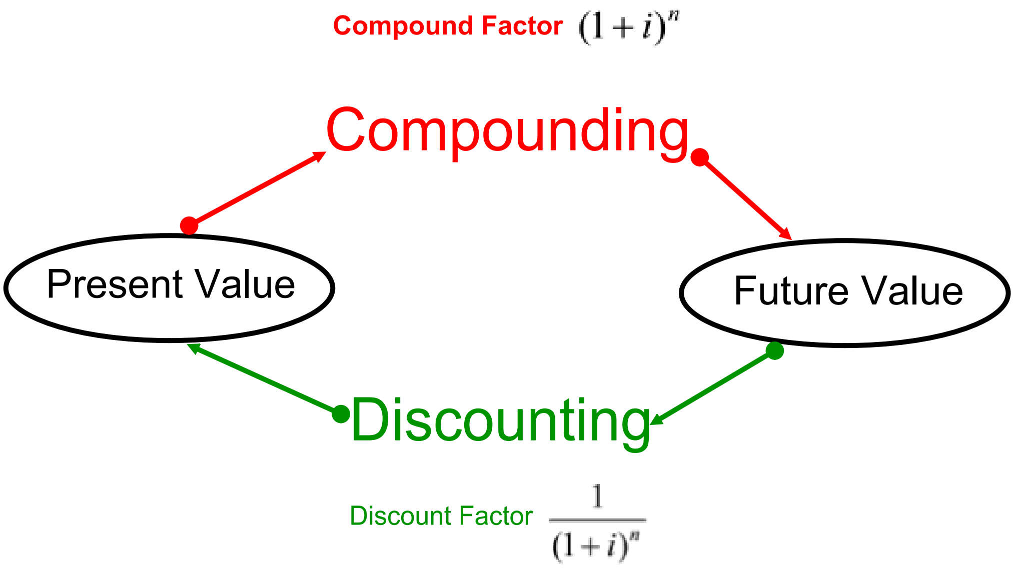
$$FV \cdot \frac{1}{(1+i)^n} = PV$$

Present Value Formula

EXCEL CLASS EXAMPLE: Use Excel to calculate the following present values:

Desired Future Value $50,000
Annual Interest Rate is 5%
Compare periods of 1 year, 2 years, 5 years, 10 years, & 20 years

Compounding Vs. Discounting



Notice that the first two functions of a dollar translate:

A lump sum future value to present value

OR

A lump sum present value to a future value.

The last four functions consider annuities. Annuities are a series of payments. In the case of the functions of a dollar, we consider a series of all equal payments.

And Now the Formulas!!!!!

Future Value Of An Annuity

$$FV = PMT \bullet \left(\frac{(1+i)^n - 1}{i} \right)$$

$FV =$ *Future Value*

$PMT =$ *Payment*

$i =$ *Interest Rate (eg Monthly or Annual)*

$n =$ *Number of periods (eg months or years)*

EXCEL CLASS EXAMPLE: Use Excel to calculate the following future values:

Compare saving $2,000 per year, $5,000 per year, and $10,000 per year
Annual Interest Rate is 6%
Consider a periods of 30 years.

Present Value Of An Annuity

$$PV = PMT \bullet \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right)$$

$FV =$

$PMT =$

$i =$ Monthly or Annual)

$n =$ Number of periods (eg months or years)

Notice that if you divide the formula for Future Value of an Annuity by the discount factor ($\frac{1}{(1+i)^n}$), one derives the formula for Present Value of an Annuity.

ALWAYS be sure to match the interest rate period, and payment period with that used in the number of periods. In other words, do not use an annual interest rate with a monthly payment.

EXCEL CLASS EXAMPLE: Use Excel to calculate the following present values:

Compare a loan with an annual payment of $2,000 per year, $5,000 per year, and $10,000 per year.
Annual Interest Rate is 6%
Consider a loan term of 30 years.

THEN Confirm the Future Value of the Annuities (see previous page) divided by the discount factor equals the Present Value of the Annuities (calculated above),

$$\frac{1}{(1+i)^n}$$

Amortization Of Future Value
AKA: Sinking Fund Factor

Start With Future Value of Annuity Formula $FV = PMT \bullet \left(\frac{(1+i)^n - 1}{i} \right)$

Multiply by reciprocal of ( ) $\bullet \left(\frac{i}{(1+i)^n - 1} \right)$ $\bullet \left(\frac{i}{(1+i)^n - 1} \right)$

$$FV \bullet \left(\frac{i}{(1+i)^n - 1} \right) = PMT$$

Amortization of Future Value Formula

EXCEL CLASS EXAMPLE: Use Excel to calculate the following amortization of future values (aka sinking fund factor):

Start with a saving goal (future value) of $100,000
Annual Interest Rate is 6%
Compare results for periods of 5 years, 10 years, and 15 years.

Amortization Of Present Value

AKA: Partial Payment Factor

Start With Present Value of Annuity Formula $PV = PMT \bullet \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right)$

Multiply by reciprocal of ( ) $\bullet \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right)$ $\bullet \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right)$

$$PV \bullet \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) = PMT$$

Amortization of Future Value Formula

EXCEL CLASS EXAMPLE: Use Excel to calculate the following amortization of present values (aka partial payment factor):

Start with a loan amount (present value) of $300,000
Annual Interest Rate is 6%
Compare results for periods of 10 years,15 years and 30 years.

Summary of Formulas
Six Functions of a Dollar

$PV =$ *Present Value*

$FV =$ *Future Value*

$PMT =$ *Payment*

$i =$ *Interest Rate (eg Monthly or Annual)*

$n =$ *Number of periods (eg months or years)*

Future Value using Compound Interest

$$FV = PV(1+i)^n$$

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$$FV \bullet \frac{1}{(1+i)^n} = PV$$

Future Value of an Annuity

$$FV = PMT \bullet \left(\frac{(1+i)^n - 1}{i} \right)$$

Present Value of an Annuity

$$PV = PMT \bullet \left(\frac{(1+i)^n - 1}{i(1+i)^n} \right)$$

Amortization of Future Value (aka Sinking Fund Factor)

$$FV \bullet \left(\frac{i}{(1+i)^n - 1} \right) = PMT$$

Amortization of Present Value (aka Partial Payment Factor)

$$PV \bullet \left(\frac{i(1+i)^n}{(1+i)^n - 1} \right) = PMT$$